Fast Product Importance Sampling of Environment Maps

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Figure 1: Our product sampling technique (top row) compared to BSDF and environment map sampling combined with MIS (bottom row) (64 samples per pixel). Our method also takes advantage of MIS, but we avoid generating light samples outside the main BSDF lobe, resulting in a big improvement, particularly for medium roughness values.

CCS CONCEPTS
• Computing methodologies → Ray tracing;

KEYWORDS
illumination, ray tracing, image based lighting, sampling

ACM Reference format:

1 OVERVIEW

Environment maps have been used for decades in production path-tracers to recreate ambient lighting conditions captured from real world scenes. Stochastic sampling of the radiance integral can be very challenging however, as both the BSDF and the environment can have strong peaks that are not aligned with each other. Multiple importance sampling (MIS) between the environment and the BSDF is a common way to reduce variance by re-weighting each estimator, but can still result in wasted samples. Product importance sampling is an effective way to reduce the variance by drawing samples using a probability distribution built from the product of the BSDF and the environment map. To our knowledge, the most practical product sampling technique [Clarberg and Akenine-Möller 2008] is still relatively costly for production rendering because it approximates the BSDF by a sparse quad-tree built on the fly from a few hundred BSDF samples. Due to the high complexity of the multi-lobed models used in film rendering, this cost can be prohibitive.

We tackle this issue by introducing an inexpensive BSDF proxy representation and replace quad-trees with a simpler two-level tabulation of the sphere. This enables product importance sampling without extra memory usage. The product of the inexpensive BSDF proxy with the environment map is evaluated on a very small 12 x 12 table corresponding to the upper level of the environment’s importance map. The result is a Cumulative Distribution Function (CDF) that only samples the relevant portions of the high resolution importance map. We show an important variance reduction with very little performance penalty on production scenes.

2 TWO-LEVEL IMPORTANCE TABLE

Our approach is built upon a two-level hierarchy of tables representing the sphere of directions. We use an area preserving spherical parameterization [Clarberg 2008] (Figure 2) to avoid distortion near the poles. This makes better use of the table cells as they all subtend the same solid angle on the sphere, which is particularly important for the upper, low resolution level of our importance table.

Figure 2: Clarberg’s equal-area square to sphere mapping, where a 12x12 table is sufficient to capture rough lobe features from BSDFs.

The bottom level captures the fine details in the map at high resolution. There is an independent high resolution table for each of the top level cells. Each of these tables covers a subsection of the sphere, and can be used to generate rays only toward those areas (while still importance sampling that portion of the environment map). The upper-level covers the whole sphere and uses a coarse resolution. We draw samples over the sphere of directions by first picking a sample from the coarse map, and then another from the high resolution table corresponding to the chosen cell.
The key advantage of this representation is that the upper level CDF is coarse enough to be rebuilt on the fly for every shading point, while the bottom level is precomputed for the whole render. This gives us the ability to steer samples as a function of the surface normal like in [Subr and Arvo 2007] but also using the BSDF.

![Upper level vs. Bottom level](image)

**Figure 3:** The upper level is a $12 \times 12$ table covering the sphere where we compute the BSDF and environment map product. The final sample comes from a finer table in the chosen cell.

### 3 BSDF PROXY REPRESENTATION

We need to compute a value for each cell of the $12 \times 12$ coarse CDF quickly in a way that will capture the important features of the BSDF. Direct BSDF evaluation can be quite expensive because representing complex materials involves multiple lobe evaluations.

A proxy for the BSDF speeds up this process. We take advantage of the fact that almost all of our BSDF lobes revolve around four different vectors: the normal, opposite normal, reflected and refracted ray. All these directions are implicit from the macro-surface normal and don’t need to be stored, only their weights and average roughnesses. Together with the IOR (which locates the refracted ray) we need a total of 7 floats to represent a complex BSDF.

A simplified GGX distribution is applied to the reflected and refracted directions; we ignore the half vector transform to speed up execution. The lobe is evaluated as $\omega = \alpha |N \cdot \omega_o|$, where $\omega$ is the total accumulated weight on the lobe and $\alpha$ is the weighted average of the roughnesses scaled by the square root of the half transform Jacobian for the reflection and refraction directions. That is $\alpha_\ell = \alpha_r = \sqrt{4 |N \cdot \omega_o|}$ and $\alpha_\ell = \alpha_r |N \cdot T| \eta + |N \cdot \omega_o|)/\sqrt{|N \cdot T| \eta^2}$. For the diffuse and translucent components, a simple $\pm \cos \theta$ term suffices.

![Lobe Directions](image)

**Figure 4:** Our proxy reduces all existing lobes to four weights plus weighted averages of all roughnesses for the reflection and refraction lobes. Each of the four axes forms some angle $\theta$ with the cone of a sky cell, used for proxy evaluation.

Our BSDF proxy is just an approximation, so we still rely on MIS to improve robustness. We always use the full BSDF representation for BSDF importance sampling which provides a fallback for cases where the BSDF exhibits high anisotropy or very low roughness, for example.

### 4 PRODUCT IMPORTANCE SAMPLING

After executing the surface shader and before lighting begins, the returned BSDFs accumulate weights to our simplified proxy. Figure 4 depicts how we use each of the four lobe directions to get an angle towards each cell in the coarse table.

The $12 \times 12$ CDF table is built by estimating the product of environment map importance and BSDF proxy for every cell. Instead of considering the cell’s center for evaluation, we choose the direction closest to the lobe axis within a safety cone surrounding the cell. This maximizes the BSDF proxy evaluation and enforces the generation of samples in regions close to the lobe falloff. It also simplifies the partial visibility computations by always providing a direction in the correct hemisphere. This step can be easily vectorized using SIMD intrinsics.

### 5 RESULTS

We show a comparison to naive MIS sampling between the environment map importance and the BSDF in Figure 5. There is a slight ~2% time increase that vanishes as scene and shading complexity grow towards production cases, and which is sufficiently justified by the reduction in variance.

![Metallic Material](image)

**Figure 5:** A metallic material under an environment map with several hot spots (100 samples/pixel). The insets compare naive MIS sampling (left) to our product importance sampling (right).

### REFERENCES

